

# Knots, Fractals and Balls.

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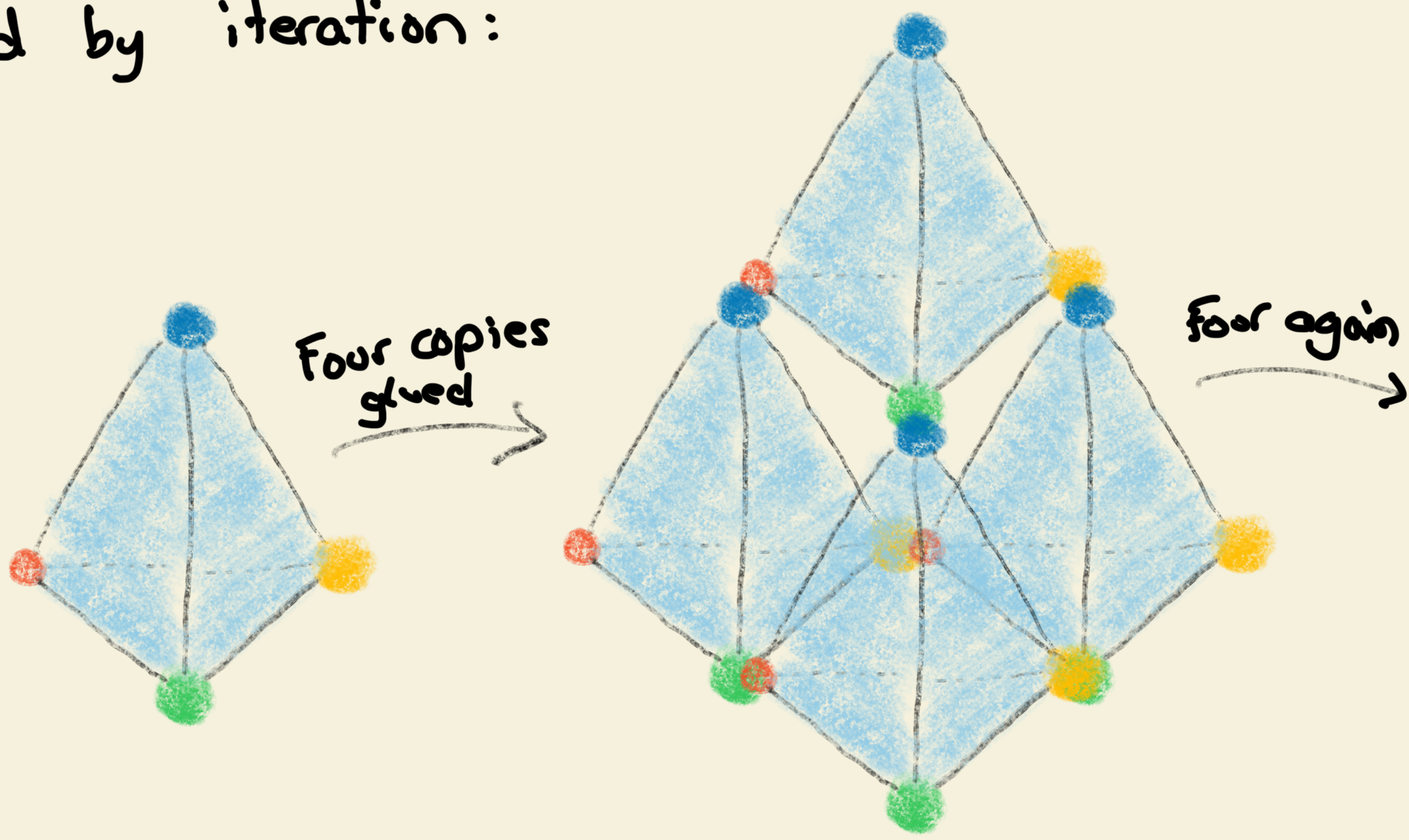
Beijing, Chi

These days that I see a lot of my friends in semi depression out of A.I. and its terrible shenanigans, I thought on sharing with you the cartoon version of what I'm writing. Just to cheer you up and help you remember that math continues to hold beauty

(I also need to test my new iPad Bowers, because this thing is expensive and to get this app was tricky.)



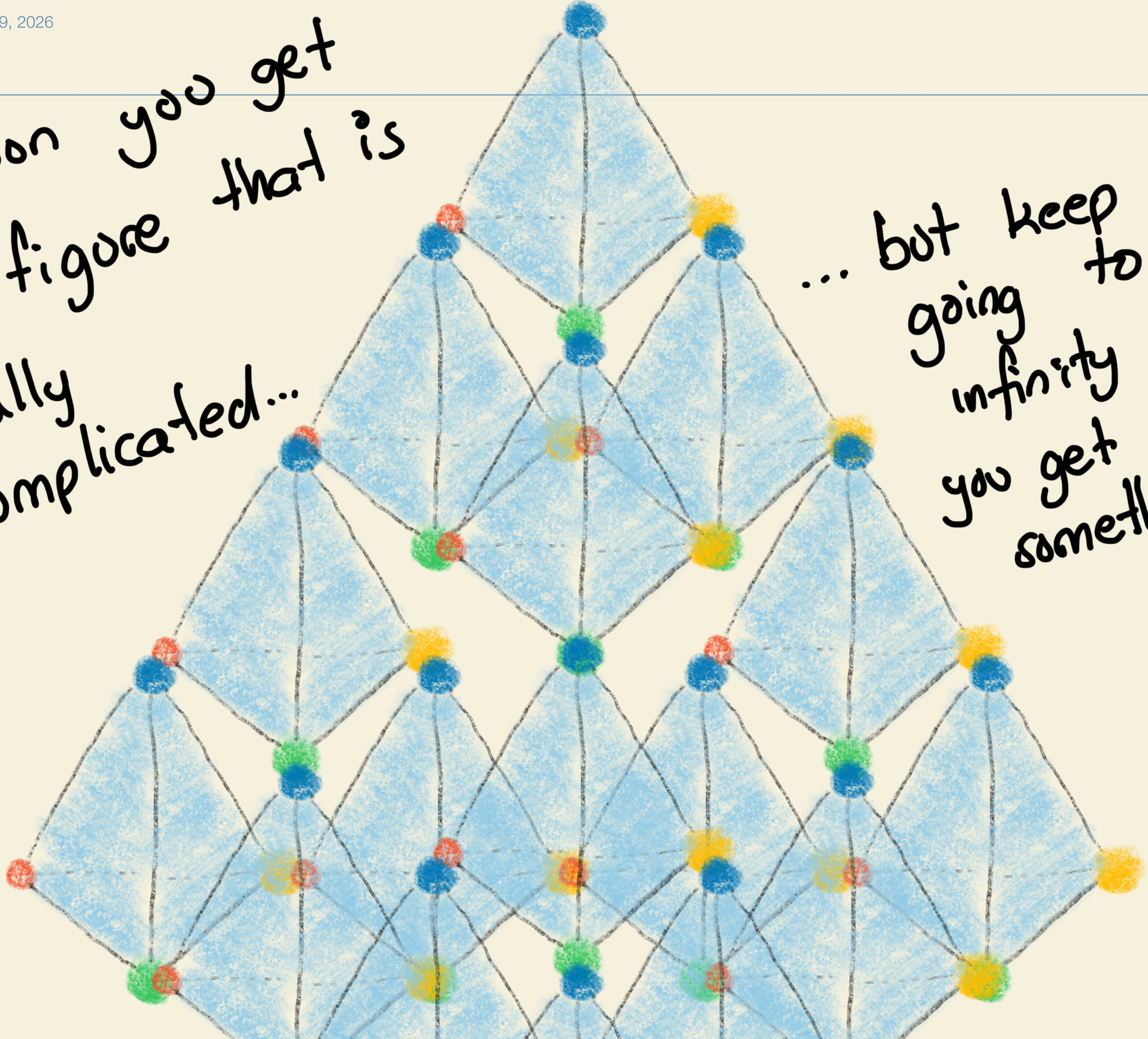
The Sierpinski Tetrahedron is a fractal that you build by iteration:





Soon you get  
a figure that is  
really complicated...

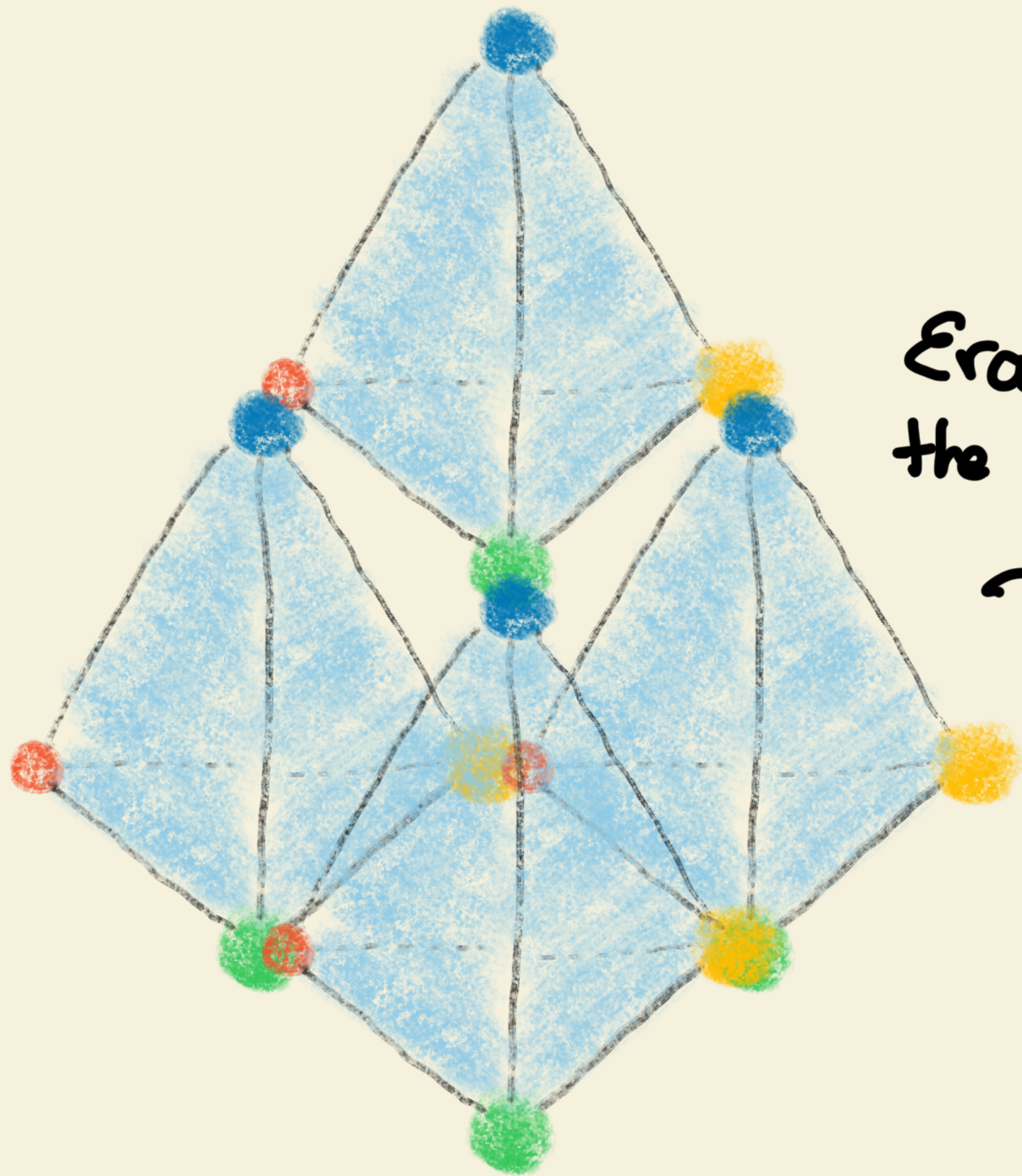
... but keep  
going  
to  
infinity and  
you get  
something.





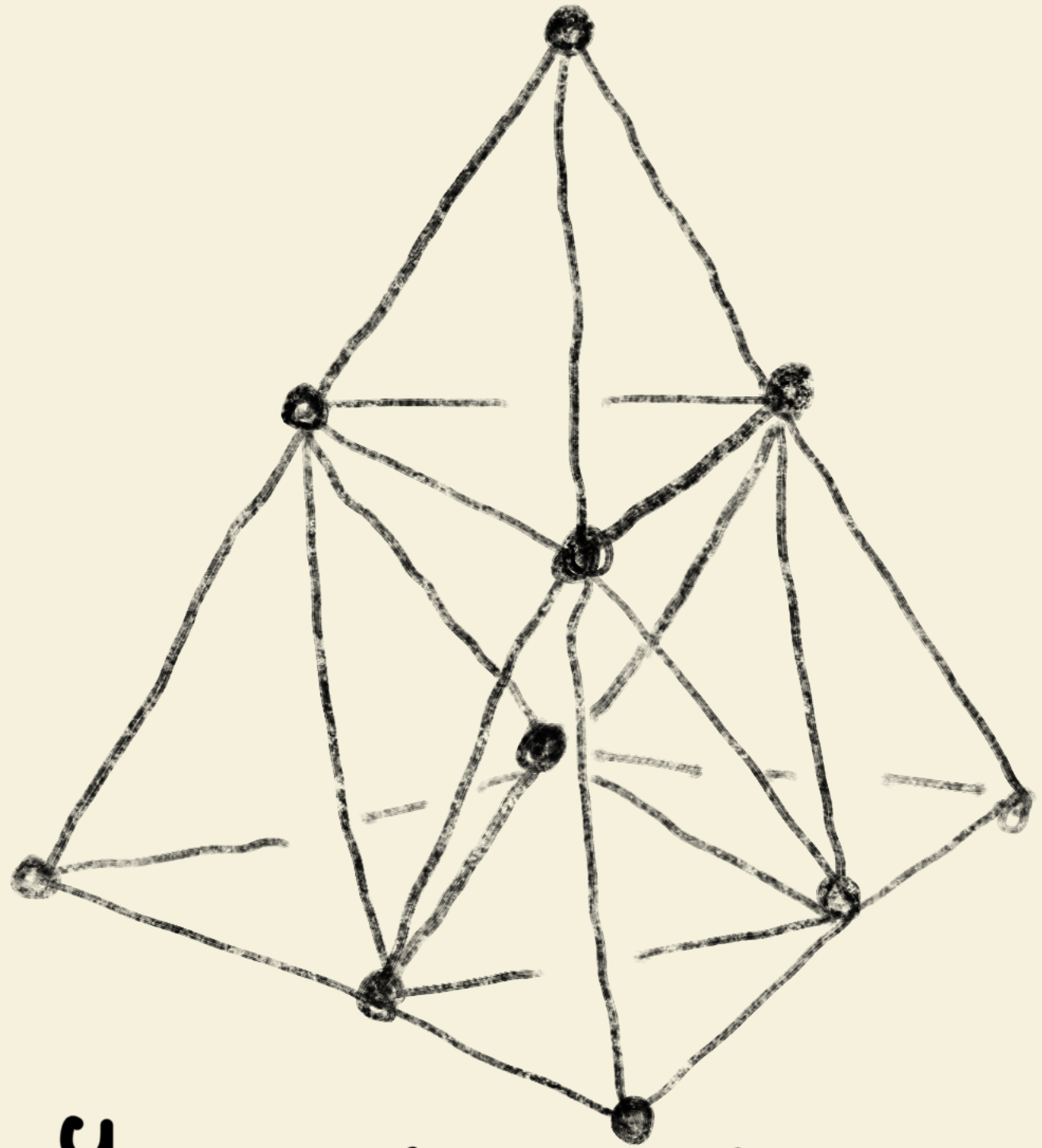
What you get at the end is a fractal called the Sierpinski Tetrahedron. With some collaborators (Josh, Noah, Niko) we looked for knots on this fractal.

The fractal is quite complicated so we simplified the search into the edges of the previous figures.



Erase  
the faces.

→

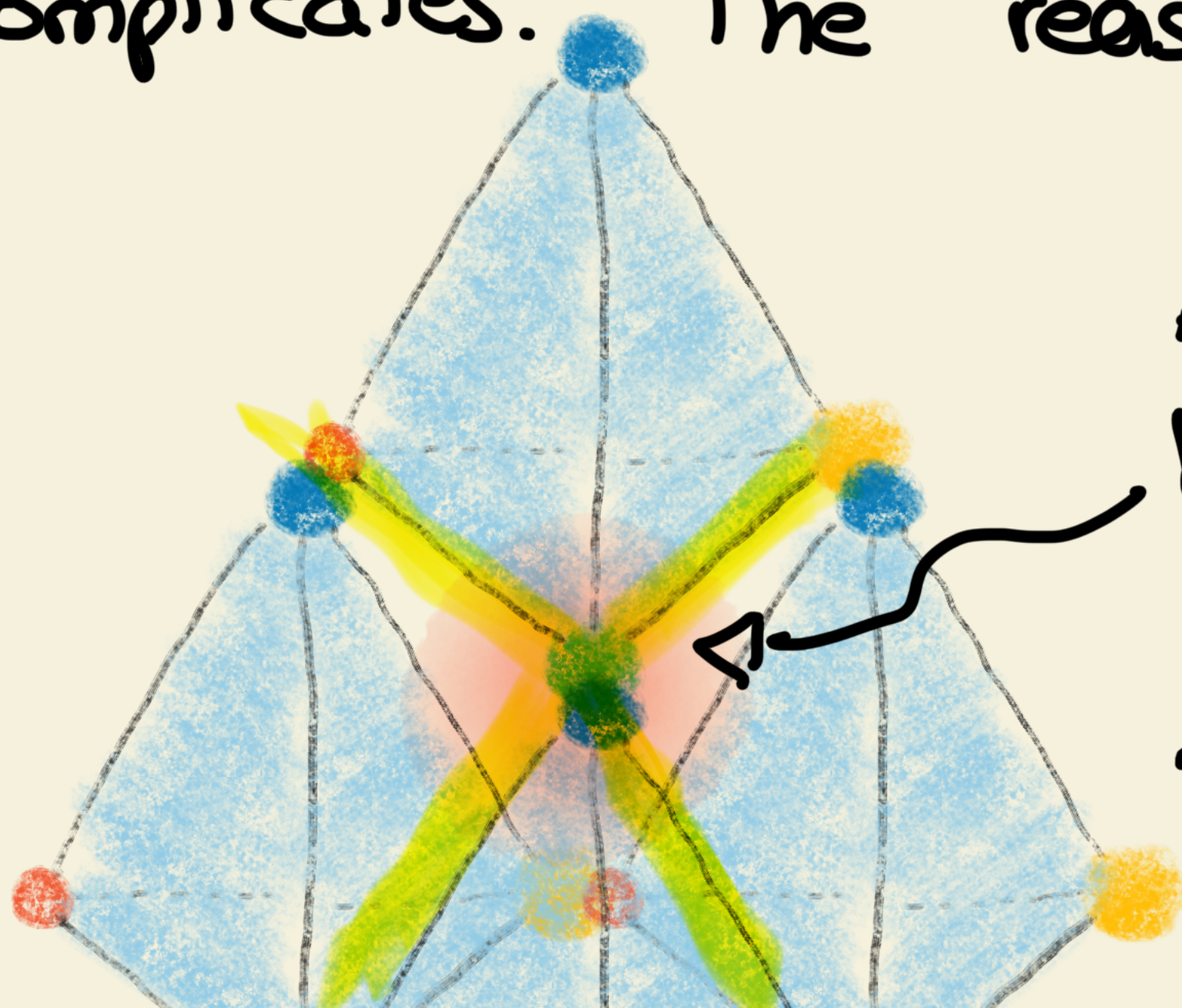


You get complicated  
graphs where you can look  
for knots.



We conjectured that as the graphs get more complicated you would find all knots eventually.

However as you look for these knots the search complicates. The reason is simple.



All vertices have very little paths for the knot to travel by.

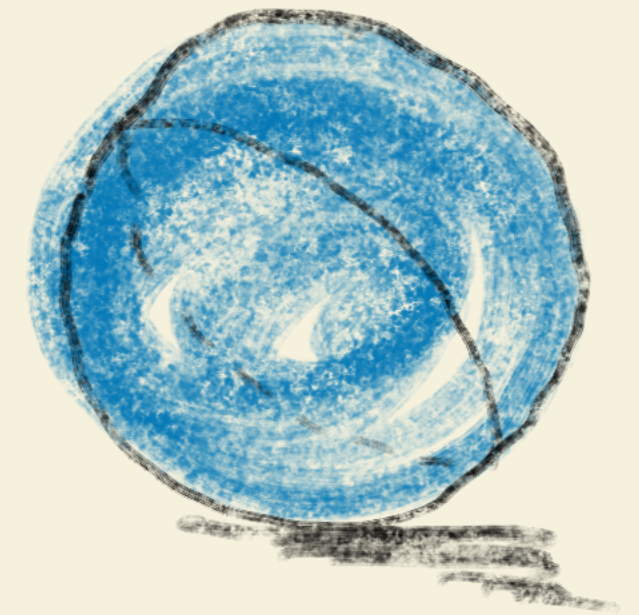


Thus a knot can only visit each pyramid twice because the vertices run out. Hence, the knot has to spread out through the pyramids, it cannot concentrate anywhere because no pyramid has enough entry points (it has four!).

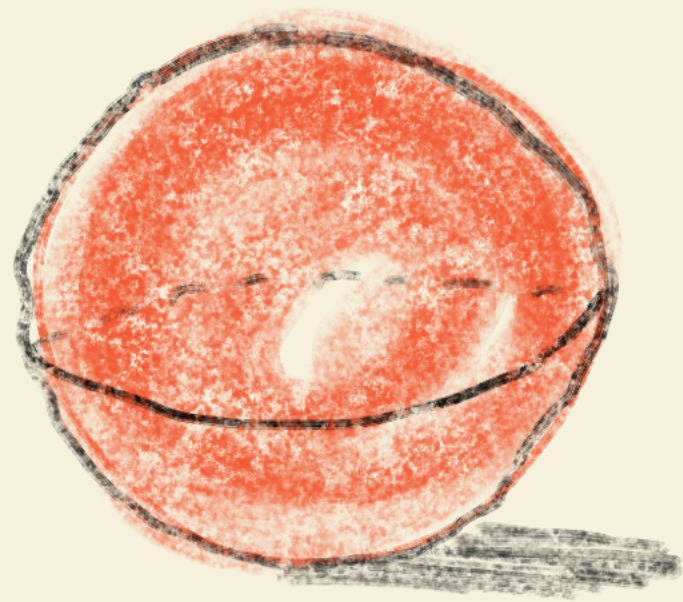
But that is an intuition, how to formalize such ideas?



Well...



turns out it

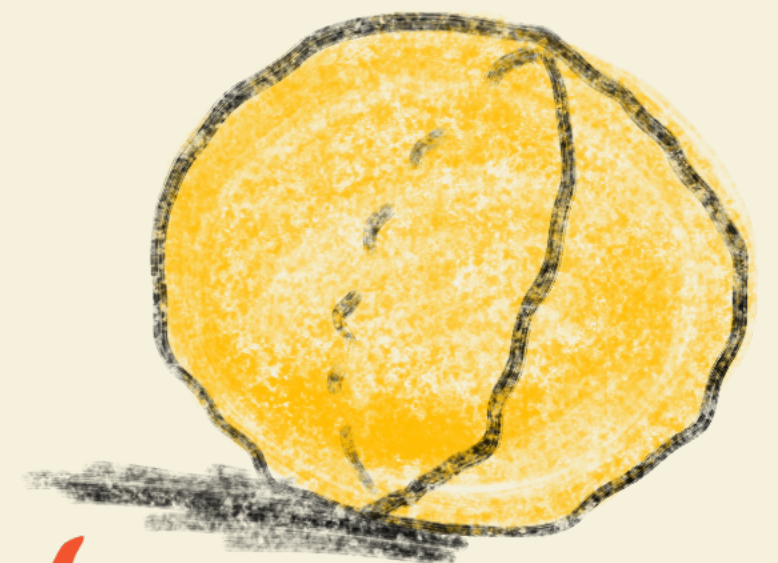


has been



done...

It is called



Sphere Decompositions.

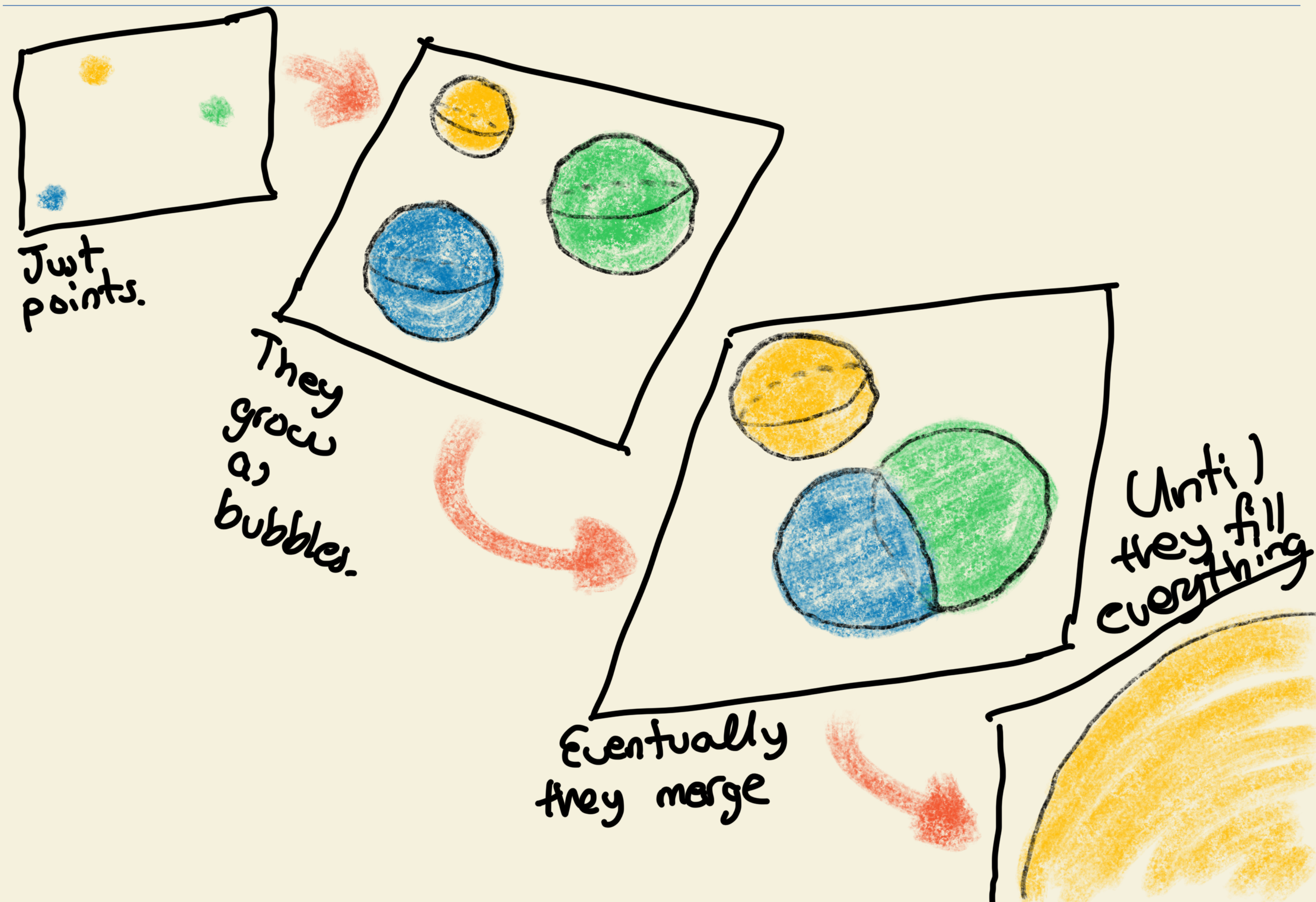


A sphere decomposition of a knot is a technical thing but what you must imagine is a movie:

Once upon a time in the  
Sierpinski Tetrahedron:

The main characters are a set of points flying somewhere and then the movie starts and they begin to grow as bubbles!!







"And what about the knot", you say?

Well the knot is just lying there while the bubbles grow.

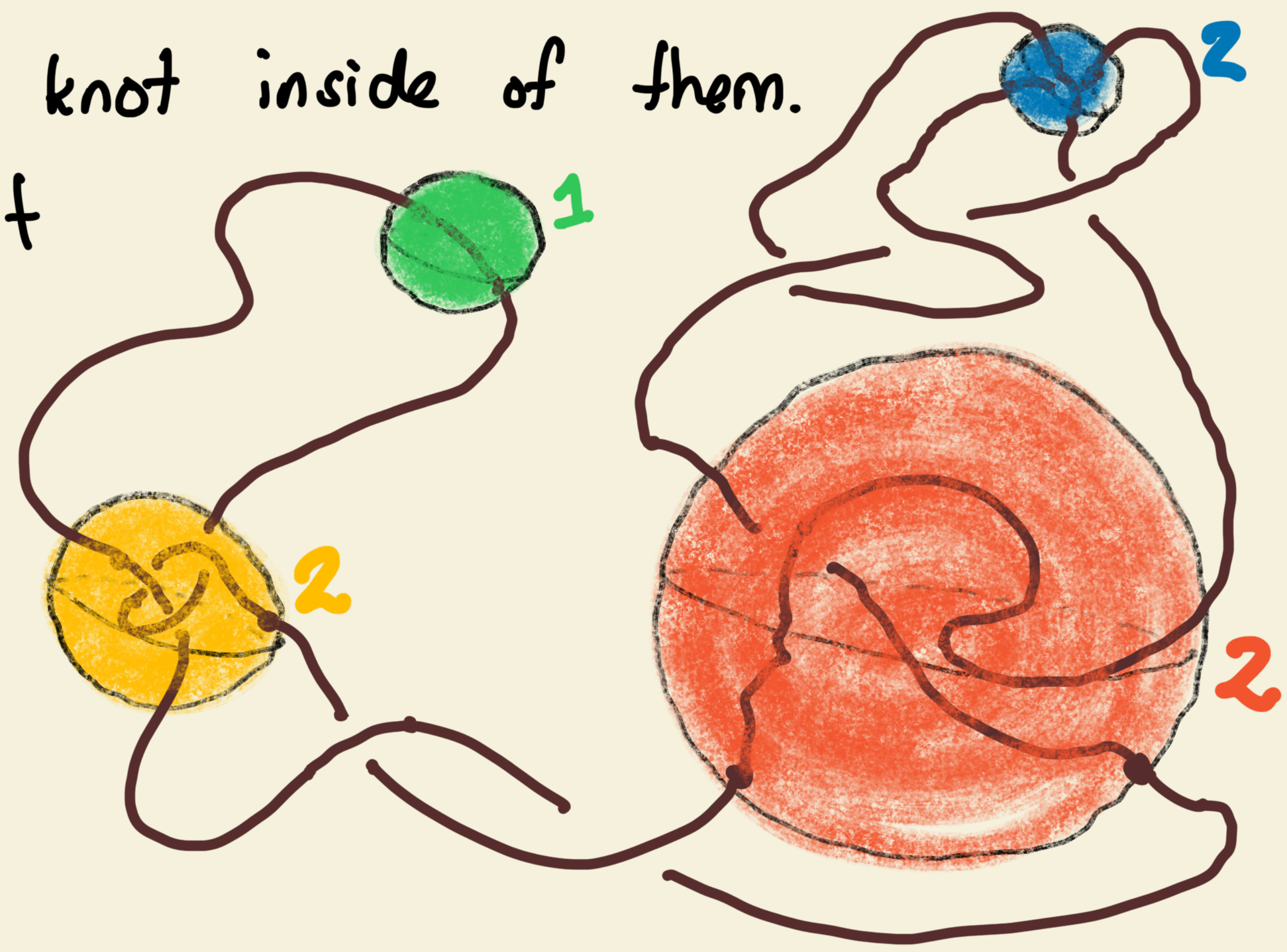


As the bubbles grow they trap more and more of the knot until it is completely trapped in the final bubble!!



If at some point of the movie you stop what you see are a bunch of bubbles with pieces of the knot inside of them.

We count how many pieces of the knot each ball has.

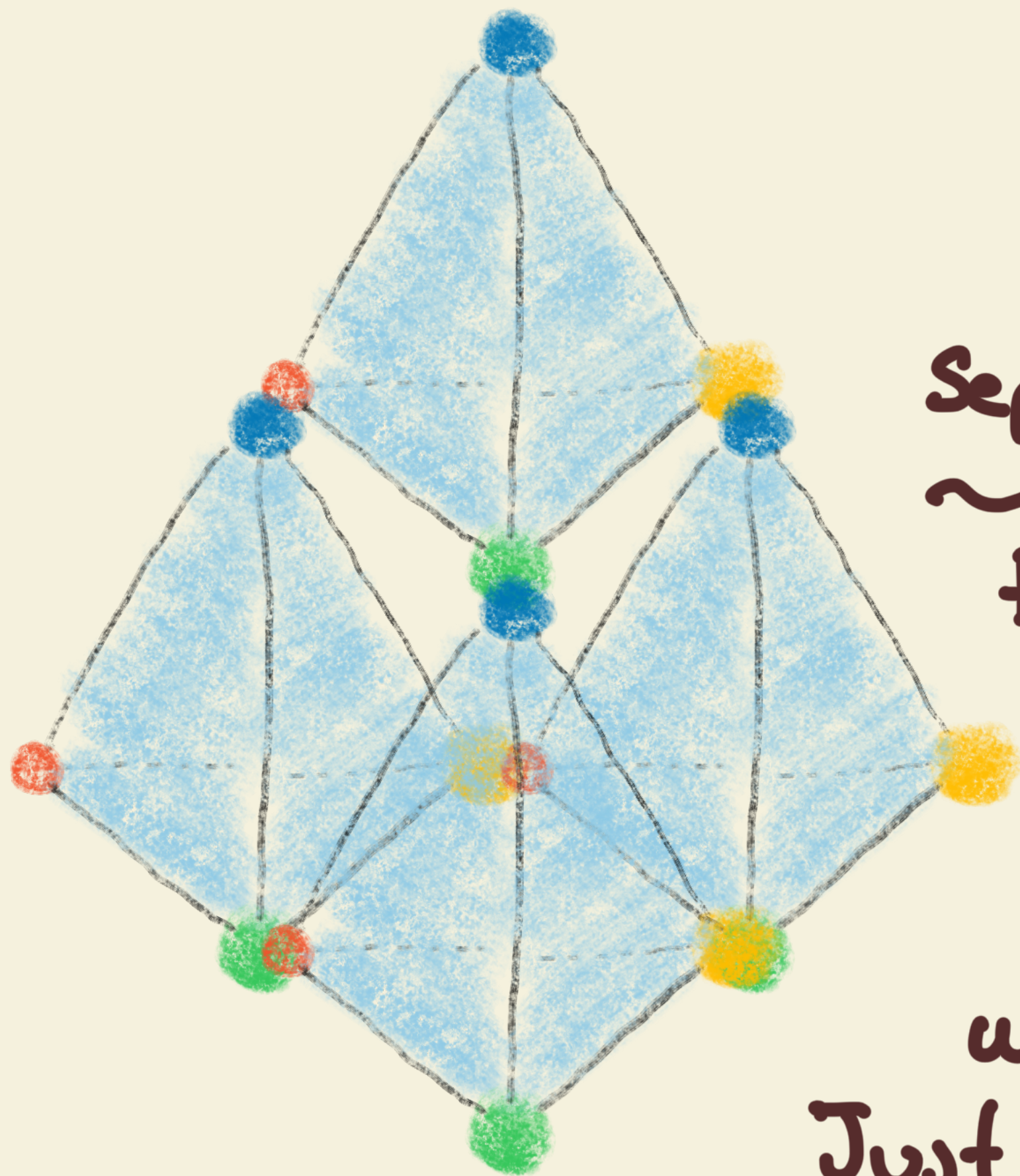




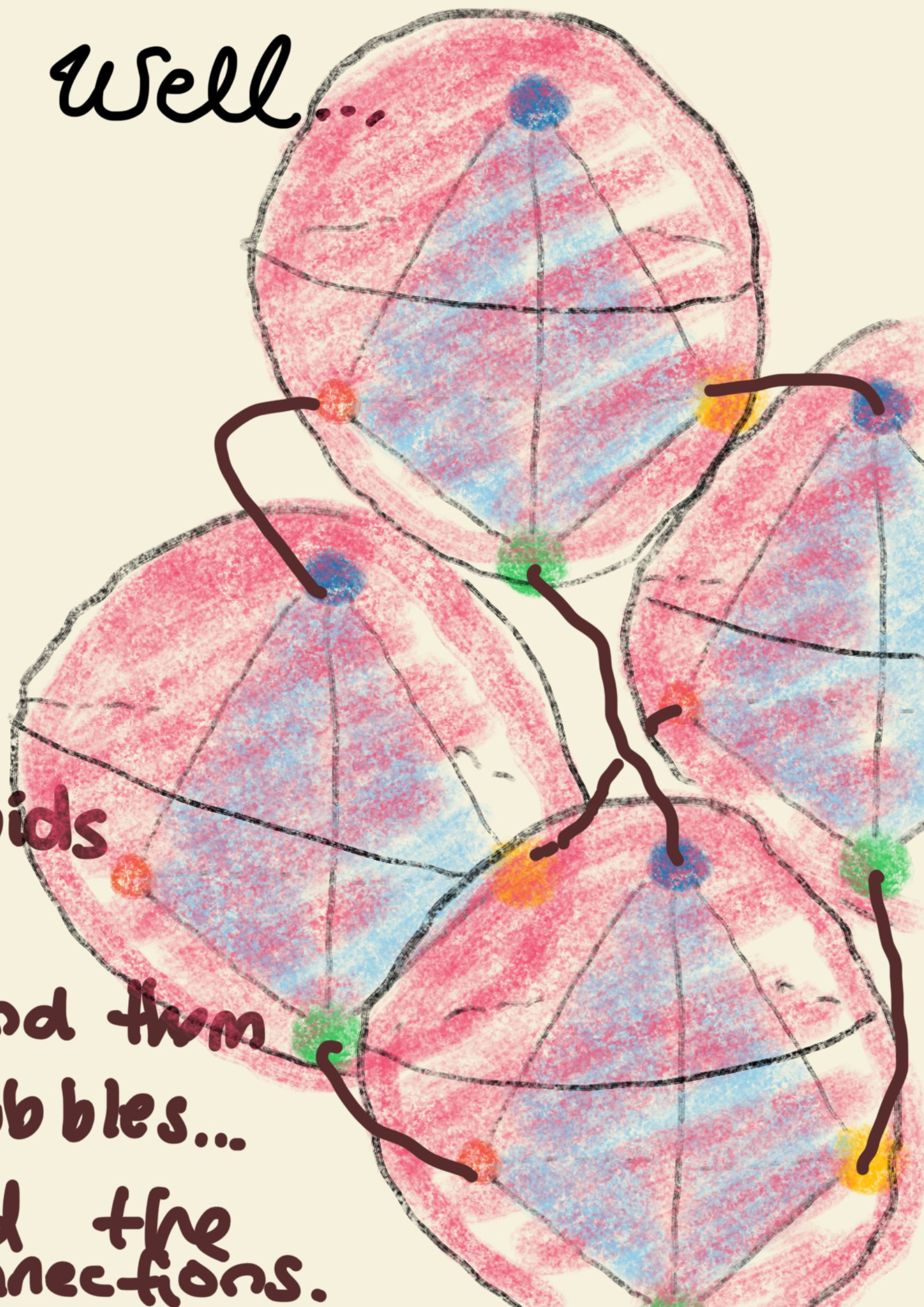
Of course, this has to be done carefully. However, the fact is that each knot has associated a complexity: this is a number  $n$  and it means that **A knot has complexity  $n$  if, in any sphere decomposition, at some point some bubble had  $n$  pieces of the knot in it.**



And what does this have to do with  
the Gierpinski Graphs? Well...

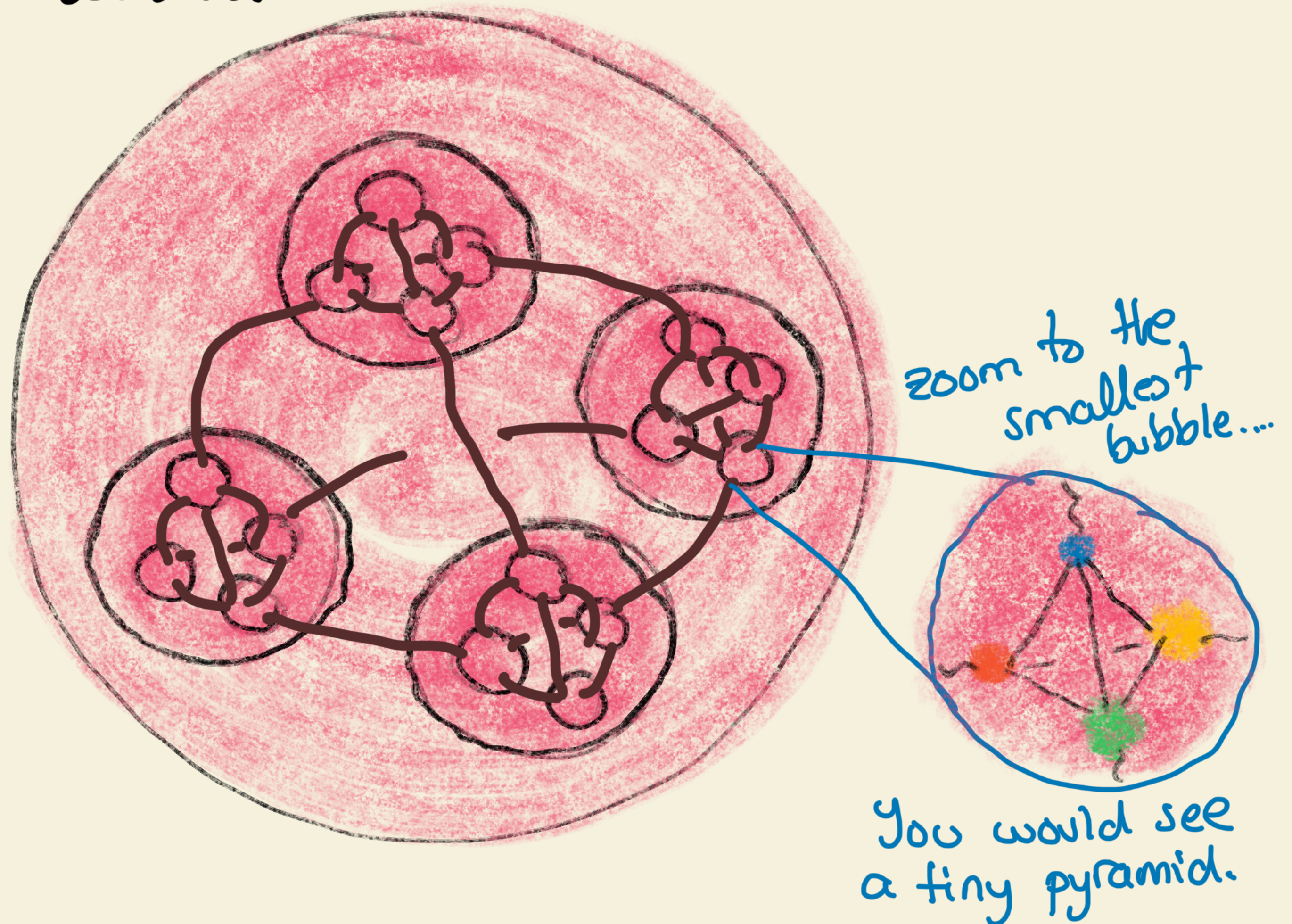


Separate  
the  
pyramids  
and  
surround them  
with bubbles...  
Just record the  
connections.





Thus your configuration of pyramids is now bubbles inside of bubbles.





Thus a finite Sierpinski graph gives you  
Snapshots of the movie.

The challenge is to prove this can be  
completed to a whole movie!!

Even more so: remember each pyramid can  
only be visited twice by the knot.

↳ Thus each sphere has  
two pieces of the knot.  
Thus the complexity  
of the knot cannot  
go very far!!



I have been able to prove with a lot of patience that all this happens and the complexity is bounded by  $4!!$

(i.e. the spheres never have more than 4 pieces.)

However...

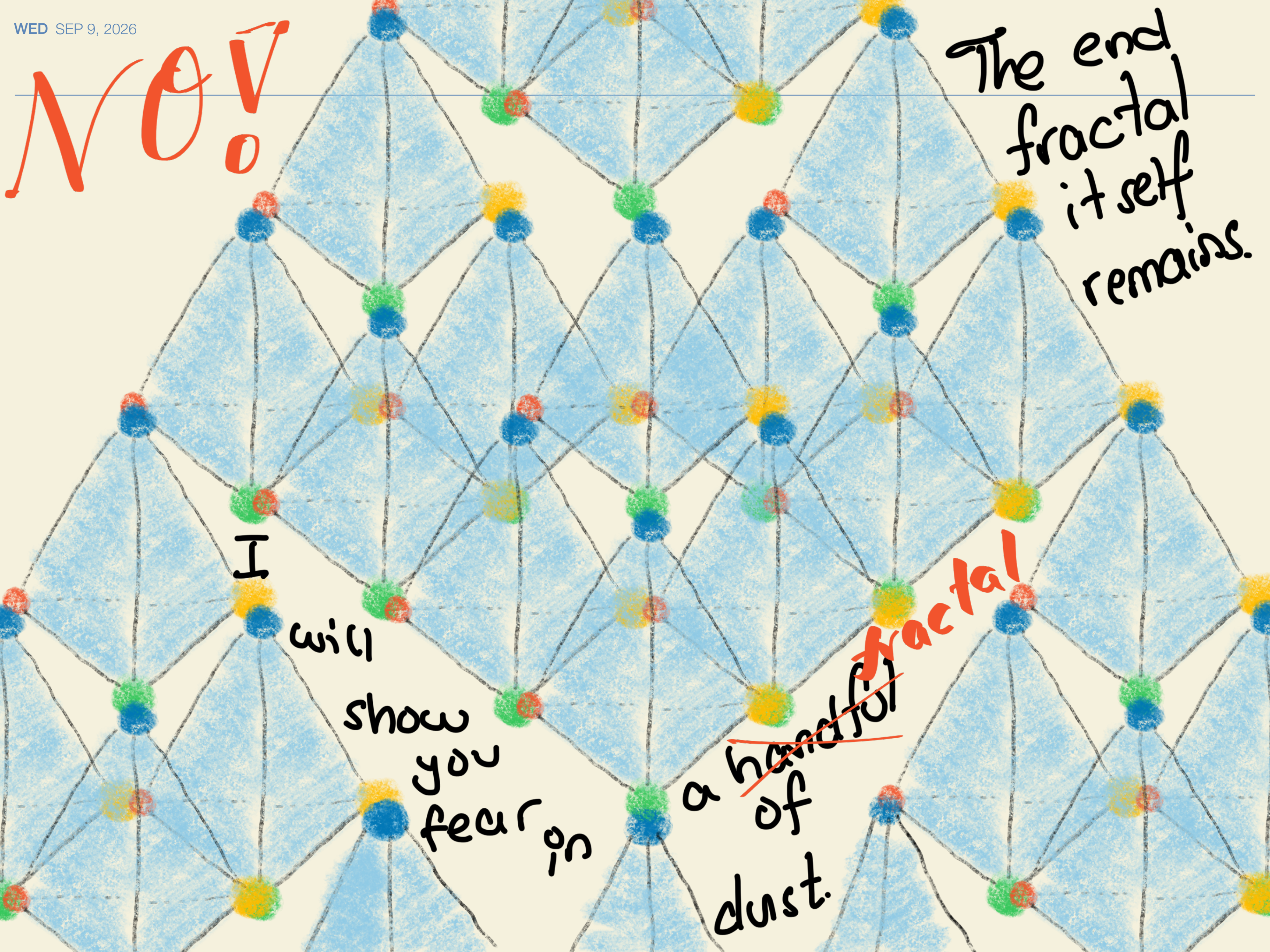
However... It is known there are knots with complexity as large as desired.



As a conclusion, there  
are knots that cannot be  
found in the finite  
stages!!

Does this conclude  
the problem?





NOV

The end  
fractal  
it self  
remains.

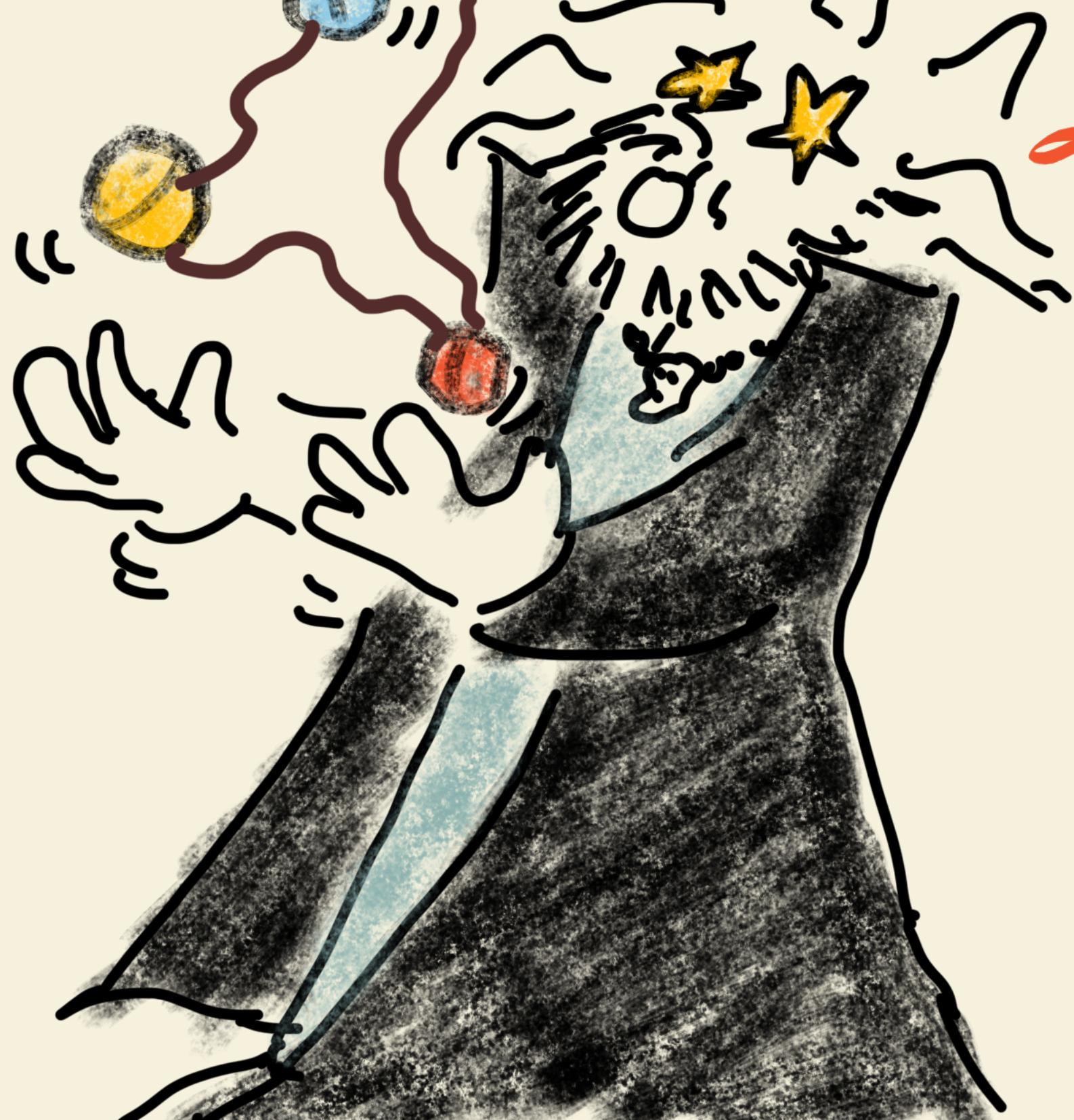
I  
will  
show  
you  
fear  
in  
a ~~handful~~  
of  
dust.

fractal



The results do not generalize. At least  
I don't know how... fractals can be  
counterintuitive.

The End



Cheer up!  
It's easier to imagine  
the end of math than  
the ruling of  
A.I.  
That's possibly a  
trap.